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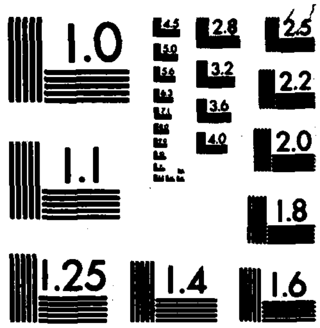

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Robust estimation of a random signal is considered when the signal to be estimated is a linear-filtered version of a source signal, which is observed after passage through a distorting channel which also adds random noise. The signal is allowed to be a vector in general. These results extend earlier results on the scalar case obtained without consideration of channel distortion.

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ROBUST WIENER FILTERING FOR MULTIPLE INPUTS WITH CHANNEL DISTORTION

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ABSTRACT

Robust Wiener filtering has previously been considered for the single-input (scalar) case where there is no channel distortion and where the signal to be estimated is the source signal itself. In this correspondence, ~~we~~ the authors extend these results to the multiple-input (vector) case where linear channel distortion is allowed and the signal to be estimated is a linear-filtered version of the source signal. The results are obtained from those for the single-input case by modifying appropriately the constraints on signal and noise characteristics. Such a modification is motivated by an examination of the expression of the mean-square error for the optimum filter.

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I. INTRODUCTION

In minimum mean-squared-error estimation of random signals observed in random noise, classical theory is applicable for obtaining optimum estimators provided signal and noise second-order characteristics are known. When these characteristics can only be confined a priori to belong to some broader classes of possibilities, minimax robust estimators may be used to obtain performance which is always at least as good as an optimum lower bound. In [1] a minimax robust Wiener smoother was obtained for the case where the scalar observation process was an additive mixture of the desired random signal and random noise. The result in [1] was obtained for signal and noise power spectral density classes of total-power-constrained spectra lying between given upper and lower bounds. In [2], this work was extended to more general classes of allowable spectra, and causal estimation filters were also considered. Another extension of the results of [1] has recently been obtained [3], applicable for cases where signal and noise are possibly correlated. It is also of relevance to note that the robust Wiener filtering results obtained in [1] are closely related to robust hypothesis testing results obtained in [4] for bounded classes of probability density functions. This relationship was observed in [2]. In fact, the results in [4] give more details of the results and proofs in [1].

In this correspondence we obtain another useful extension of the basic robust Wiener smoother result in [1]. We consider here the case where a linear channel (e.g. measuring instrument or transmission channel) distorts the original signal s(t) whose power spectral density is modeled as belonging to a bounded class, so that the observation process is the

sum of a distorted version of the original signal and noise. In addition, we will look for estimates of a linear-filtered version d(t) of s(t). In this work we will assume that the channel distortion and the linear filter generating d(t) from s(t) are specified. The case where the channel distortion is imprecisely specified but signal and noise spectra are known has also been recently considered [5]. In addition, we simultaneously allow our signal, noise and observation processes to be vector-valued, for which we use spectral models which are direct extensions of those used in the scalar case.

The results that we establish for the robust filters in our more general case can be obtained by modifying the proofs of the results obtained for the simple case considered in [1,4]. Although we do not attempt it here, it should be possible to include signal and noise correlation (as in [3]) in this more general scheme, and also to apply the restriction to causal filters [2].

II. PROBLEM STATEMENT

Consider the model of an m-input signal estimation problem in Figure 1. Here $\underline{s}(t)$ is the original signal m-component real vector process, and it is desired to obtain an estimate $\hat{\underline{d}}(t)$ of a desired k-vector $\underline{d}(t)$ which is a linearly filtered version of $\underline{s}(t)$. Specifically, we have

$$d(t) = (K^T * \underline{s})(t), \quad -\infty<t<\infty, \tag{1}$$

where $K^T(t)$ is a k x m real matrix and the "*" denotes matrix-vector convolution. The observation process $\underline{y}(t)$ is a real m-component vector

process given by

$$\underline{y}(t) = (C^T * \underline{s})(t) + \underline{n}(t), \quad -\infty<t<\infty, \tag{2}$$

where the m x m real matrix-valued function C^T is the observation-channel impulse-response and $\underline{n}(t)$ is a real m-component noise process. We assume that the processes $\underline{s}(t)$ and $\underline{n}(t)$ are uncorrelated, zero-mean and wide-sense stationary, with respective power spectral density matrices[1] $S(\omega)$ and $N(\omega)$. Further, we assume that the Fourier transforms $K(\omega)$ and $C(\omega)$ of $K(t)$ and $C(t)$, respectively, exist and are known. Let $H^T(\omega)$ be the k x m frequency response matrix of a linear filter (not necessarily causal) used for estimating $\underline{d}(t)$. We require $H(\omega)$ to belong to the class $\mathcal{H}$ of frequency response matrices of real filters. Then the resulting mean-squared-error $e(H; S, N) \triangleq E\{[\underline{d}(t) - \hat{\underline{d}}(t)]^T[\underline{d}(t)-\hat{\underline{d}}(t)]\}$, where $\hat{\underline{d}}(t)$ is the estimate, can be shown to be given by

$$e(H; S, N) = \text{trace } \{\frac{1}{2\pi}\int_{-\infty}^{\infty} [K(\omega) - C(\omega)\, H(\omega)]^{\dagger} \cdot S(\omega)[K(\omega) - C(\omega)\, H(\omega)] + H^{\dagger}(\omega)\, N(\omega)\, H(\omega)\, d\omega\}, \tag{3}$$

where "†" denotes conjugate transpose.

Our problem is to find a minimax robust filter characterized by frequency response matrix $H_R(\omega)$, for specific classes $\mathcal{S}$ and $\mathcal{N}$ of allowable spectral density matrices $S(\omega)$ and $N(\omega)$, respectively, so that

$$\min_{H\in\mathcal{H}} \max_{\substack{S\in\mathcal{S}\\ N\in\mathcal{N}}} e(H, S, N) = \max_{\substack{S\in\mathcal{S}\\ N\in\mathcal{N}}} e(H_R, S, N) \tag{4}$$

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[1]We define the autocorrelation matrix of $\underline{y}(t)$ as $E\{\underline{y}(t)\underline{y}^T(t+\tau)\}$.

The specific classes S and N we will consider are generalizations to the matrix case of the bounded classes of power-constrained spectral densities considered in [1, 2], and for these a minimax robust filter will be found which satisfies

$$\min_{H \in H} \max_{\substack{S \in S \\ N \in N}} e(H, S, N) = \max_{\substack{S \in S \\ N \in N}} \min_{H \in H} e(H, S, N)$$

$$= e(H_R, S_R, N_R). \tag{5}$$

In this case H_R is the frequency response of the optimum filter corresponding to the pair $(S_R, N_R) \in S \times N$, which is called the <u>least-favorable</u> pair.

III. ASSUMPTIONS AND ALLOWABLE CHARACTERISTICS

We assume that the channel characteristic $C(\omega)$ is of the form $C_0(\omega)I$, where $C_0(\omega)$ is a scalar function and I denotes the identity matrix. This assumption holds for the case where the source signals all pass through the same observation channel and do not interfere with each other in the observation channel.

Since $S(\omega)$ and $N(\omega)$ are hermitian, non-negative definite matrices, they can be decomposed into $P_s(\omega)\Lambda_s(\omega)P_s^{\dagger}(\omega)$ and $P_n(\omega)\Lambda_n(\omega)P_n^{\dagger}(\omega)$, respectively, where $P_s(\omega)$ and $P_n(\omega)$ are unitary matrices consisting of the normalized eigenvectors of $S(\omega)$ and $N(\omega)$, and $\Lambda_s(\omega)$, $\Lambda_n(\omega)$ are diagonal matrices consisting of the eigenvalues of $S(\omega)$ and $N(\omega)$, respectively. We will assume that $P_s(\omega) = P_n(\omega) = P(\omega)$. This assumption, although restrictive, does hold for some practical situations. For example, if the noise components are independent of each other and each have the same power spectral density, (i.e., $N(\omega)$ is diagonal matrix with the same diagonal terms), the columns of $P(\omega)$ can be set to be the normalized eigenvectors

of $S(\omega)$. We also assume that $P(\omega)$ is known, so that uncertainty about $S(\omega)$ and $N(\omega)$ is with respect to the eigenvalue functions $\Lambda_s(\omega)$ and $\Lambda_n(\omega)$. In a practical application of these results one would start with a nominal description for S and N which are diagonalizable by the same matrix P , and then allow deviations from the nominal in the diagonal terms. While this assumption again restricts the applicability of the results, it is necessary for obtaining the explicit results that we give. Several interesting examples can be given in which such restrictions hold [6]. This assumption implies that the m components of the observation process $\underline{y}(t)$ can be decoupled prior to further processing.

Let $\lambda_{si}(\omega)$ and $\lambda_{ni}(\omega)$ denote the i-th diagonal terms of $\Lambda_s(\omega)$ and $\Lambda_n(\omega)$. For allowable $S(\omega)(=P(\omega)\Lambda_s(\omega)P^\dagger(\omega))$ and $N(\omega)(=P(\omega)\Lambda_n(\omega)P^\dagger(\omega))$, we assume that $\Lambda_s(\omega)$ and $\Lambda_n(\omega)$ satisfy the following constraints:

$$\lambda_{s\ell i}(\omega) \le \lambda_{si}(\omega) \le \lambda_{sui}(\omega) \tag{6a}$$

$$\sum_{i=1}^{M} \int_{-\infty}^{\infty} A_i(\omega)\ \lambda_{si}(\omega)d\omega = 2\pi Q_s \tag{6b}$$

and

$$\lambda_{n\ell i}(\omega) \le \lambda_{ni}(\omega) \le \lambda_{nui}(\omega) \tag{7a}$$

$$\sum_{i=1}^{M} \int_{\Omega} A_i(\omega)\ \lambda_{ni}(\omega)/|C_0(\omega)|^2\ d\omega = 2\pi Q_n \tag{7b}$$

where $A_i(\omega)$ is the i-th diagonal term of $P^\dagger(\omega)K(\omega)K^\dagger(\omega)P(\omega)$, and Ω denotes the set of all ω such that $C_0(\omega) \neq 0$. Note that (6b) is a power constraint on the desired signal $\underline{d}(t)$, since the left-hand side of (6b) can be shown to be $\operatorname{tr} \int K^\dagger(\omega)S(\omega)K(\omega)d\omega$. Similarly, (7b) is a power constraint on the

noise which would appear if the estimated desired signal is obtained directly by passing $\underline{y}(t)$ through a linear filter with frequency response $K^T(\omega)/C_0(\omega)$, the "inverse" filter. Thus, the constraints (6) and (7) may be specified by a priori information.

In the following, we will find the robust solution for the allowable $S(\omega)$ and $N(\omega)$ just defined. As will be seen, the results can be obtained by a generalization of the proof of the results in [1, 2].

IV. SOLUTION FOR ROBUST FILTER

Let S_o, N_o be any pair of spectral density matrices in the classes $\mathcal{S}$, $\mathcal{N}$, respectively, defined in Section III. Then the optimum filter frequency response $H_o^T(\omega)$ for this pair is given by

$$H_o(\omega) = [|C_0(\omega)|^2 S_o(\omega) + N_o(\omega)]^{-1} C_0^{\dagger}(\omega) S_o(\omega) K(\omega); \tag{8}$$

here "$\dagger$" denotes complex conjugate. Let $\lambda_{soi}(\omega)$, $\lambda_{noi}(\omega)$ be the i-th diagonal elements of $\Lambda_{so}(\omega)$ and $\Lambda_{no}(\omega)$ in the decompositions $P(\omega)\Lambda_{so}(\omega)P^{\dagger}(\omega)$ and $P(\omega)\Lambda_{no}(\omega)P^{\dagger}(\omega)$ of $S_o(\omega)$ and $N_o(\omega)$, respectively. Using (8) in (3) and after some simple matrix manipulations, we get

$$e(H_o, S, N) = \sum_{i=1}^{M} \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\frac{\lambda_{noi}(\omega)}{|C_0(\omega)|^2 \lambda_{soi}(\omega) + \lambda_{noi}(\omega)}\right]^2 \cdot A_i(\omega)\lambda_{si}(\omega)d\omega$$

$$+ \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[\frac{|C_0(\omega)|^2 \lambda_{soi}(\omega)}{|C_0(\omega)|^2 \lambda_{soi}(\omega) + \lambda_{noi}(\omega)}\right]^2 \cdot \frac{A_i(\omega)}{|C_0(\omega)|^2} \lambda_{ni}(\omega)d\omega \tag{9}$$

Now this can be rewritten as

$$e(H_o, S, N) = \sum_{i=1}^{M} \frac{1}{2\pi} \int_{\infty}^{\infty} [1 - \tilde{H}_{oi}(\omega)]^2 A_i(\omega)\lambda_{si}(\omega)d\omega$$

$$+ \frac{1}{2\pi} \int_{-\infty}^{\infty} [\tilde{H}_{oi}(\omega)]^2 \frac{A_i(\omega)}{|C_0(\omega)|^2} \lambda_{ni}(\omega)d\omega, \tag{10}$$

where

$$\tilde{H}_{oi}(\omega) = \frac{A_i(\omega)\lambda_{soi}(\omega)}{A_i(\omega)\lambda_{soi}(\omega) + [A_i(\omega)\lambda_{noi}(\omega)/|C_0(\omega)|^2]} \tag{11}$$

Note that the optimum filter frequency response can be described by

$$H_o(\omega) = \frac{1}{C_0(\omega)} P(\omega) \tilde{H}_o(\omega) P^{\dagger}(\omega) K(\omega) \tag{12}$$

where $\tilde{H}_o(\omega)$ is the m x m diagonal matrix of elements $\tilde{H}_{oi}(\omega)$.

Defining $\tilde{\lambda}_{si}(\omega) = A_i(\omega)\lambda_{si}(\omega)$ and $\tilde{\lambda}_{ni}(\omega) = [A_i(\omega)/|C_0(\omega)|^2]\lambda_{ni}(\omega)$ we obtain the expression

$$e(H_o, S, N) = \sum_{i=1}^{M} \frac{1}{2\pi} \int_{-\infty}^{\infty} [1 - \tilde{H}_{oi}(\omega)]^2 \tilde{\lambda}_{si}(\omega)$$

$$+ [\tilde{H}_{oi}(\omega)]^2 \tilde{\lambda}_{ni}(\omega)d\omega, \tag{13}$$

This is significant because for the simple scalar problem where the desired signal power spectral density is $\sigma(\omega)$, noise power spectral density is $\eta(\omega)$ and the observation process is the desired signal plus noise, a filter with frequency response $\Gamma(\omega)$ gives a mean-squared-error between output and the desired signal component at the input of

$$\epsilon(\Gamma, \sigma, \eta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} |1-\Gamma(\omega)|^2 \sigma(\omega) + |\Gamma(\omega)|^2 \eta(\omega)d\omega \tag{14}$$

The robust Wiener filter for this problem when σ and η are in power-constrained bounded classes was obtained explicitly in [1].

We see then that in our case, in light of the definitions for our spectral density classes [Eqs. (6) and (7)], the least-favorable spectra and the robust filter may be obtained directly from the results in [1, 4], at least for the case $m = 1$. For this case ($m = 1$) we have thus shown how the presence of a non-ideal channel and an arbitrary definition for a desired signal (as some linear-filtered version of the original signal) may be included in the problem formulation for robust Wiener filtering by defining the power constraint appropriately [Eqs. (6b) and (7b)]. For the more general case $m > 1$, a simple extension of the proof for the scalar case in [1] gives the solution for the robust filter. The extension consists of simply using the more general expressions (6b) and (7b) involving summations over m components for the power constraint; the solution for the least-favorable characteristics is again obtained in terms of two constants k_s and k_n. These define $\tilde{\lambda}_{sri}(\omega)$ and $\tilde{\lambda}_{nri}(\omega)$ for the least-favorable characteristics, for each i, exactly as in the scalar case. This extension is easily proved; one way to justify it is to consider the case where in a scalar case, m disjoint frequency subsets have upper and lower spectral bounds defined, and total power constrained. As an example, if $N(\omega)$ is exactly specified and $S(\omega) = P(\omega)\Lambda_s(\omega)P^{\dagger}(\omega)$ is constrained by (6), we find

$$\lambda_{sri}(\omega) = \frac{1}{A_i(\omega)} \max\{A_i(\omega)\lambda_{s\ell i}(\omega),\ \min\{k_s A_i(\omega)\lambda_{ni}(\omega)/ |G_0(\omega)|^2,\ A_i(\omega)\lambda_{sui}(\omega)\}\} \tag{15}$$

Note that $A_i(\omega)$ may be removed everywhere it occurs in (15).

It is possible to consider a more general error expression of the form $e_Q(H, S, N) = E\{[\underline{d}(t)-\hat{\underline{d}}(t)]^T Q\ [\underline{d}(t)-\hat{\underline{d}}(t)]\}$ where Q is non-negative definite and Hermitian, and generalize the above results. In this case the $A_i(\omega)$ have to be taken as the diagonal elements of $P^\dagger(\omega)K(\omega)QK^\dagger(\omega)P(\omega)$. A complete proof of these results is given in [6].

V. SUMMARY

In this correspondence, we have extended earlier results on robust Wiener filtering which had been obtained for the scalar case when there was no channel distortion and when the signal to be estimated was the source signal itself. The choice of allowable characteristics considered here was motivated by an examination of the expression for $e(H_0, S, N)$. Although the constraints (6b) and (7b) are not put on the power spectral density matrices of the source signal and input noise directly, they are meaningful in applications. Results for the robust solution are obtained directly from the previous results [1, 4] by noting the correspondence between the roles of $A_i(\omega)\lambda_{si}(\omega)$, $A_i(\omega)\lambda_{ni}(\omega)/|C_0(\omega)|^2$ and the roles of the densities considered in [1, 4].

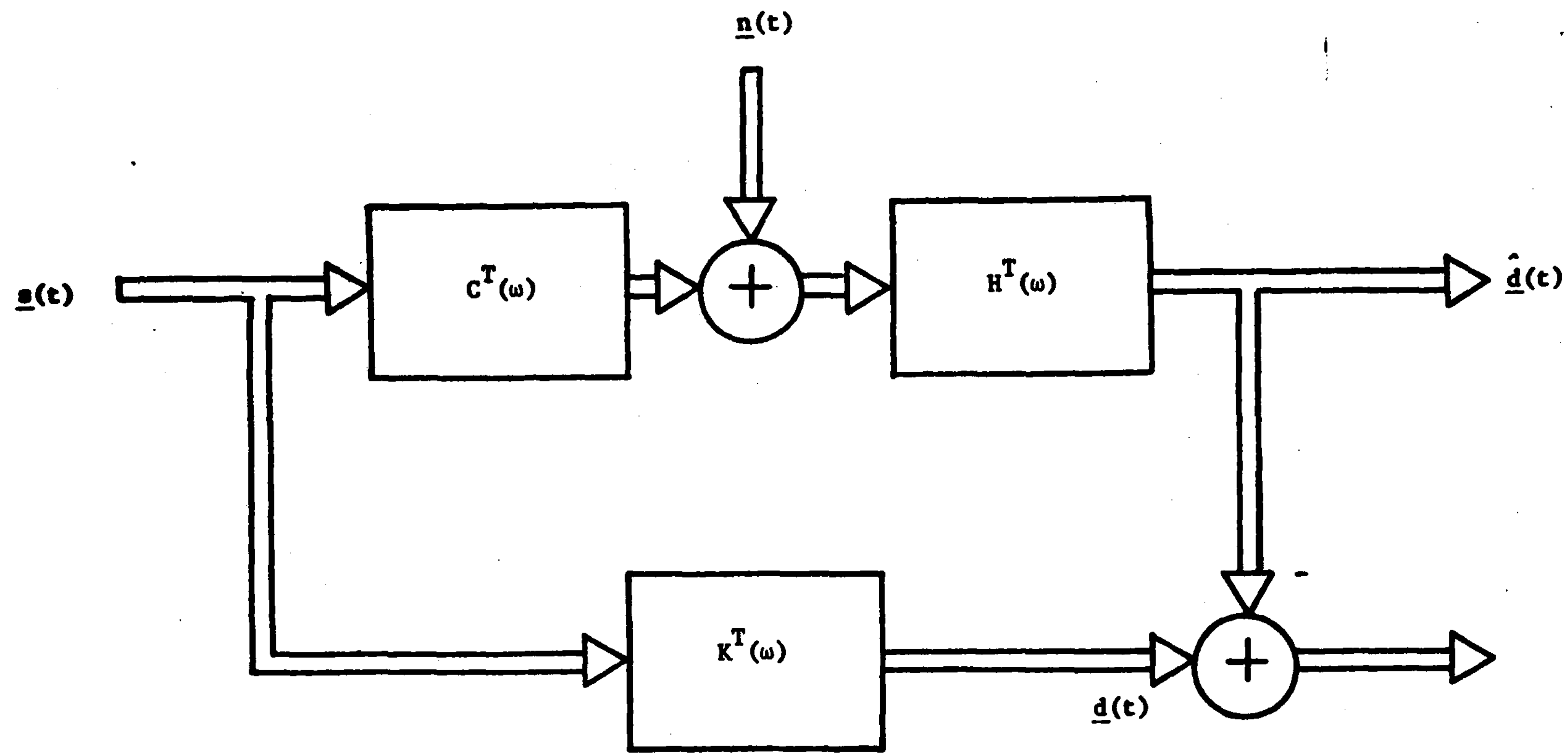

FIGURE 1

Model for Estimation Problem

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